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Comparison of Prompt Kinetics Models Derived from Alternate Eigenvalues

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INTRODUCTION

Point kinetics is a classic model to approximate transient behavior in nuclear systems using static analysis. Traditionally, the approximation is based upon a k or multiplication eigenvalue form of the neutron transport equation. Alternative, but less well known, eigenvalue formulations of the transport equation are also possible[1–4]. In this study, the collision, or c-eigenvalue, and leakage, or l-eigenvalue, forms of the transport equation are used to obtain alternate point kinetics models. The k, c and l eigenvalue equations are identical at criticality, but differ from each other for off critical systems.

Using different formulations of the point kinetics model for non-critical systems leads to different estimates of the inverse prompt period α, which are different than the true α obtained from a time-dependent calculation or measurement. The suitability of using each of α's from different point kinetics models as an approximation for the true inverse prompt period is studied. This compares them with calculated time-dependent prompt α for a few multigroup systems with representative cross sections. The result shows for a couple test simple cases of fast systems with low-Z reflectors, the prompt α obtained from c eigenvalue kinetics may be more representative than the k eigenvalue. It appears the l eigenvalue kinetics does not agree with time-dependent prompt α and is most likely of little practical use.

THEORY

The k-eigenvalue equation is

$$(L + T - S)\,\psi_k = \frac{1}{k} F \psi_k. \tag{1}$$

Here L is the operator for streaming, T is the total interaction operator, S is the integral scattering operator, and F is the integral total fission operator. This equation forms a balance relation that finds the correction factor or eigenvalue k that balances the left- and right-hand sides of the equation along with the corresponding eigenfunction ψ_k.

There is nothing sacred about applying the correction factor to the multiplication term. Alternatively, a factor c could be applied to both the scattering and fission terms to obtain

$$(L + T)\,\psi_c = \frac{1}{c}\,(S + F)\,\psi_c. \tag{2}$$

The factor c increases the multiplicity of all collisions, as opposed to just fission, to balance the equation. This form has been studied before, primarily by mathematicians trying to understand the properties of the transport equation. As an aside, unlike the k or l eigenvalues, c, like α, exists even in the absence of fissionable material.

Another, rarely seen, form of the transport equation is

$$L\psi_l = \frac{1}{l}\,(S + F - T)\,\psi_l \tag{3}$$

that balances the streaming operator with the other terms. The leakage eigenvalue l can be thought of as a factor to globally adjust the atomic density to achieve criticality. Interestingly, there are many cases where no density increase may achieve criticality, and in these cases the l eigenvalue does not exist.

The equations are all identical at criticality ($k = c = l = 1$) and therefore have the same fundamental eigenfunctions; however, when the system is not critical the eigenvalues and eigenfunctions differ.

None of these equations involve time dependence, which, in general, must be obtained by solving the full, time-dependent neutron transport equation,

$$\frac{1}{v}\frac{\partial \psi}{\partial t} = (S + M - L - T)\psi + \sum_i \lambda_i C_i(t) + Q, \tag{4}$$

the calculation of which may be rather difficult. Here the prompt fission operator M is separated from the delayed neutron precursors C_i explicity. The instantaneous inverse period is defined as the logarithmic time derivative of the neutron population

$$\alpha(t) = \frac{1}{N(t)}\frac{dN(t)}{dt}. \tag{5}$$

Alternatively, often asymptotic behavior of the prompt neutrons is only of interest, which may be obtained from the α-eigenvalue form of the transport equation:

$$(S + M - L - T)\,\psi_\alpha = \frac{\alpha}{v}\psi_\alpha. \tag{6}$$

The α eigenvalue can be thought of as an additive insertion of a $1/v$ absorber for a supercritical system or source for the subcritical case to balance the equation. This makes sense as in the former case, the slower neutrons cannot keep up with the exponentially rising neutron population, and in the latter, it is those slower neutrons that are driving the

exponentially falling population. For late times and in the absence of sources or delayed neutrons, the α eigenvalue and the inverse period (and flux shape) are identical. The α-eigenvalue form of the transport equation is usually easier to solve for supercritical systems; however, the stability of numeric methods for subcritical systems makes the α eigenvalue difficult to obtain as well. For this study, the direct, time-dependent approach is used.

These calculational difficulties may be avoided if somehow the static eigenvalue models may be connected to time dependence. This may be done with the point kinetics model. To obtain this, the time-dependent neutron flux is assumed to be in the fundamental mode and separable in time and the rest of phase space,

$$\psi(\mathbf{r}, \hat{\mathbf{\Omega}}, E, t) = n(t)\varphi(\mathbf{r}, \hat{\mathbf{\Omega}}, E). \tag{7}$$

From here, a static eigenvalue form of the adjoint transport equation is employed with eigenfunction $\psi_x^\dagger$, where $x = k$, c, or l. The separable time-dependent transport equation is multiplied by the adjoint function, the adjoint equation is multiplied by the shape function φ, both are integrated over all space and subtracted. After some manipulation, the point kinetics equation is obtained

$$\frac{dn}{dt} = \left(\frac{\rho_x - \beta_x}{\Lambda_x}\right) n(t) + \sum_i \lambda_i c_i(t) + q. \tag{8}$$

The precursor equations are not written here, but must be solved as well. The terms ρ_x, β_x, and Λ_x are called the reactivity, effective delayed neutron fraction, and effective generation time respectively. The exact form of these depend upon the assumption of static eigenfunction chosen, hence the x subscript. The point kinetics inverse prompt period α_x is given by,

$$\alpha_x = \frac{\rho_x - \beta_x}{\Lambda_x}, \tag{9}$$

which can be used as an approximation for the true asymptotic prompt α from the time-dependent analysis.

The reactivity is defined as

$$\rho_x = 1 - \frac{1}{x}. \tag{10}$$

The β_x and Λ_x are

$$\beta_x = \frac{\left\langle \psi_x^\dagger, (F - M)\varphi \right\rangle}{\left\langle \psi_k^\dagger, A_x \varphi \right\rangle}. \tag{11}$$

$$\Lambda_x = \frac{\left\langle \psi_x^\dagger, \frac{1}{v}\varphi \right\rangle}{\left\langle \psi_k^\dagger, A_x \varphi \right\rangle}. \tag{12}$$

Here A_x is a generic operator, which is F for the k-eigenvalue kinetics, $S + F$ for the c-eigenvalue kinetics, and $S + F - T$ for the l-eigenvalue kinetics.

The shape function φ is for the time-dependent transport equation, and is typically unknown unless the time-dependent or α problem is solved, which defeats the purpose of employing point kinetics. Often the shape function is approximated by the static forward eigenfunction being used, typically ψ_k. The most logical choice is to use the forward eigenfunction consistent with the adjoint eigenfunction selected. Strictly speaking, this is not necessary and any representative function can be used, and therefore different forward and adjoint eigenfunctions may be mixed. For this work, only the consistently chosen forward and adjoint functions are used.

Different choices of eigenfunction lead to different values and physical significance of the kinetics parameters; however, at criticality (and only at criticality), the combinations of terms as kinetics α_x are identical and equal to the inverse prompt period α. This also implies that the transition from delayed to prompt supercritical is predicted differently depending on the point kinetics model chosen. Therefore, these point kinetics models are only useful surrogates for prompt behavior inasmuch as the kinetics α_x adequately approximates the true α.

RESULTS

For this initial study, 1-D slab geometry and multigroup cross sections are used. The systems studied were a bare metal system and a metal system with a low-Z reflector. Methods for solving the forward and adjoint equations were implemented into a research discrete ordinates (S_N) code. A research Monte Carlo (MC) code was created to solve the forward and time-dependent transport problems—time dependent S_N was developed as well, but because very small time steps and negative flux fix-ups are required, MC proved to be more robust and efficient in this case. To summarize, the kinetics parameters were obtained via S_N (S_{64} Gauss-Legendre quadrature with fine spatial mesh), and the prompt α was obtained with MC by a least-squares fit to the asymptotically changing population; time cutoffs were used in the MC to control the neutron population for the supercritical cases.

As a note of verification, the S_N and MC forward eigenvalues from the two methods match, and the k eigenvalue case was benchmarked with MCNP6.1 in multigroup mode[5], providing confidence the equations are being solved correctly by both methods. As expected, the k, c, l, and time-dependent results are identical for a critical configuration. The forward and adjoint S_N eigenvalue results for k, c, and l calculations are also identical.

Bare, 2-Group Slab

Bare slabs with varying thicknesses a form the first test case. The slab cross sections are given in Table I, and

Table I. 2-group core cross sections.

g	σ_c	σ_f	ν	χ	σ_{sg1}	σ_{sg2}
1	1.5	1.0	2.8	1.0	0.98	0.02
2	20.0	120.0	2.5	0.0	0.00	20.0

the atomic density is 0.05 atoms per barn-cm. The speeds of the two groups are $v_1 = 1.0$ and $v_2 = 0.1$ in arbitrary units. For this case, no delayed neutrons were used—the 4-group case to be discussed has them.

The slab thickness a is varied from 15 to 30 cm, with the critical thickness at about 25.5 cm. Figure 1 shows the various α_x values compared with the true (reference) prompt α. For the bare case, α_k and α_c are almost identical regardless of slab thickness, whereas α_l deviates significantly from the other two away from criticality; of course, all agree at critical.

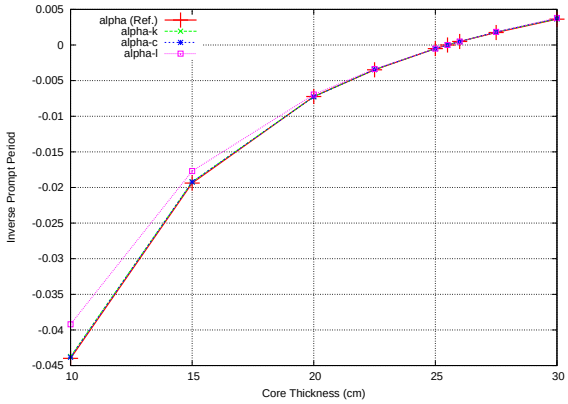

Fig. 1. True α versus kinetics α_x for Bare Slab.

Half-Reflected, 2-Group Slab

The second test case is the same core slab of thickness a with a fixed 25 cm thick low-Z reflector on the right side of the slab; the left side still has a vacuum boundary condition. The reflector cross sections are given in Table II, and, like the core, the atomic density is 0.05 atoms per barn-cm.

Figure 2 gives a comparison of the kinetics α_x to the true (reference) α. Unlike with the bare case, all three α_x are noticably different. Like before the α_k and α_c are near-

Table II. 2-group reflector cross sections.

g	σ_c	σ_f	ν	χ	σ_{sg1}	σ_{sg2}
1	0.5	0.0	0.0	0.0	2.25	2.25
2	1.0	0.0	0.0	0.0	0.00	5.0

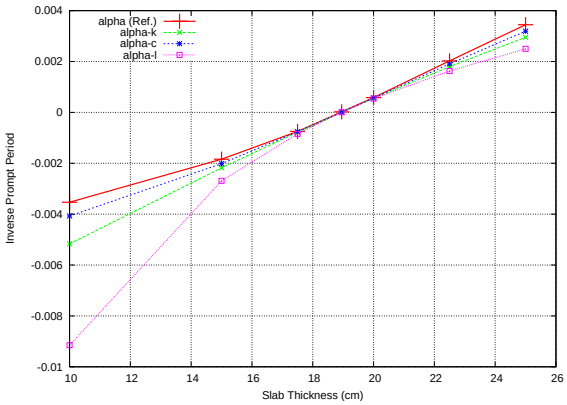

Fig. 2. True α versus kinetics α_x for Half-Reflected Slab.

est with the α_l being the noticeable outlier. In this case, all three α_x underpredict the true inverse prompt period; however, α_c appears to be the most accurate approximation over the entire range.

The normalized shape functions for the case where $a = 15$ cm are given in Fig. 3. This shows also that the scalar flux estimated by the c-eigenvalue does the best at matching the flux shape of the α-eigenvalue. In the core it overpredicts the least, and underpredicts the least in the reflector. Next accurate is the scalar flux for k with the scalar flux for l being in most disagreement.

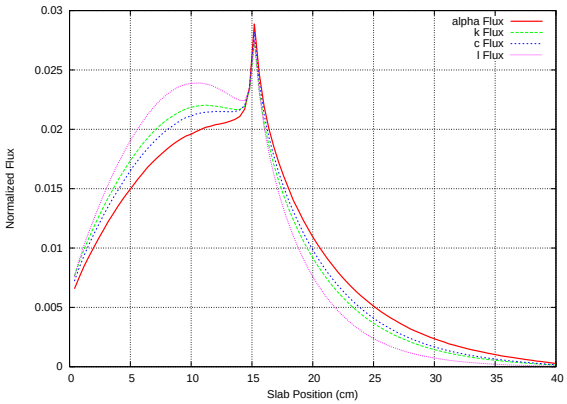

Fig. 3. Shape functions of the half-reflected case from different kinetics models.

Fully-Reflected, 4-Group Slab

The next case is reflected on both sides (modeled as a reflecting boundary condition at $x = 0$). The reflector thickness on each side is again 25 cm. The four energy groups are centered at 1 MeV, 100 keV, 10 eV, and 0.025 eV, so more realistic speeds are used in this case. Core and reflector cross sections are omitted because of space

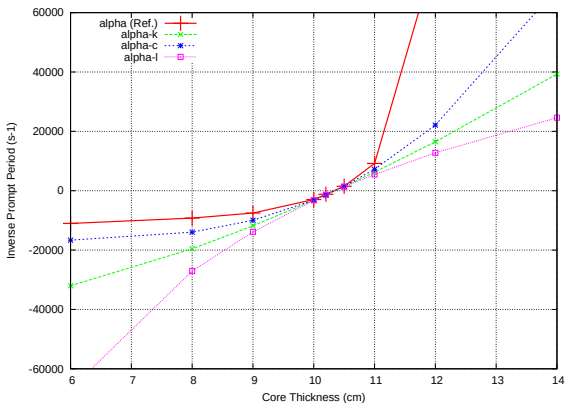

Fig. 4. True α versus kinetics α_x for Fully-Reflected Slab.

limitations, but will be published in a full paper. Delayed neutrons are included with ^{235}U data used for the β_i and λ_i. To be consistent with point kinetics, only prompt ν is used to sample the fission multiplicity in the time-dependent MC calculations.

The core thickness a is varied as with the previous case. Similar comparisons of α_x and the inverse prompt period are given in Fig. 4. None of the α_x do particularly well in predicting the inverse prompt period in this case when it is far from critical. This is because on both sides, the point kinetics model, being a perturbative approach, does not adequately insert the $1/v$ absorber or source to account for the fact that slower or faster neutrons do not significantly impact the transient. Like before, α_c is most predictive of the inverse prompt period and does better than α_k near critical; α_l is again a poor estimator of α.

SUMMARY & FUTURE WORK

Alternate versions of the point-kinetics model were derived based upon the multiplication k, collision c, and leakage l eigenvalues. Research S_N and MC codes were created to test the ability of these models to predict the inverse prompt period α obtained from a time-dependent MC simulation. Three cases with multigroup cross sections were tested: bare fast core, fast core half-reflected by low-Z material, and fast core fully-reflected by low-Z material. While it would be premature to conclude overly much based upon a few simplistic test problems using representative, but non-physical, nuclear data, the results suggest it may be worth exploring different point kinetics model, and that there may be advantages to using the c-eigenvalue kinetics for reflected systems.

The next step is to adapt these eigenvalue calculations into a continuous-energy MC code. Some of this work has already been done for the forward case with the c-eigenvalue[6]; however, new methods will need to be developed to handle the adjoint weighting needed to calculate the alternate kinetics parameters. These will most likely be logical extensions of the iterated fission probability method[7] used to compute the k point kinetics parameters.

Given the parameters from the different point kinetics models, the inhour equations may be formulated and solved. Comparisons can then be made between the models to either time dependent calculations or measurements. Finally, it may be possible to apply different combinations of these point kinetics models to multi-region kinetics, particularly regions with non-fissionable reflectors.

ACKNOWLEDGMENTS

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