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MCNP Variance Reduction Examples

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Abstract

MCNP has a fairly rich set of variance reduction techniques. These variance reduction techniques exist in MCNP because they are applicable to a wide variety of common Monte Carlo calculations. The MCNP manual describes what the variance reduction methods do in a detailed sense, but many of these details only really start making sense when the Monte Carlo practitioner has seen a few examples using variance reduction methods. This report gives a number of examples of using variance reduction methods in MCNP.

Contents

1 Introduction

Variance reduction techniques exist in MCNP[1], and many other Monte Carlo codes, because calculations cannot be done fast enough without these techniques. People tend to learn the variance reduction techniques as they need them and for their particular calculations at hand. The MCNP manual describes how the variance reduction techniques work in a detailed sense, but users often have trouble understanding if and/or how these techniques might be useful to their problem. The report "A Sample Problem for Variance Reduction in MCNP"[2] is often useful reading in this regard for MCNP users. In fact, the success of [2] gives some impetus for the current report because it indicates the value of teaching variance reduction, at least in part, by examples.

While it is hoped that the examples in this report will be useful to MCNP users, the examples constitute only part of what variance reduction is about. For some mathematically inclined users, standard books (e.g. [3], [4], and [5]) that discuss variance reduction theory mostly from an integral equation viewpoint that may be of considerable value. The reader of these books is cautioned that the variance reduction theory in these books is typically not directly applicable to most MCNP calculations that the author has seen[6]. Nonetheless, the books can impart some general understanding of variance reduction issues that may not be apparent from looking at examples.

Some of the examples that are given herein are in direct response to questions that users have raised either in MCNP courses or on the MCNP internet forum. Other examples are more in the style of [2].

2 The Weight Window Generator Output is All Zeroes

One of the most common frustrations of users results from using the weight window generator as a black box without really understanding how the generator works. The generator is a quite simple statistical estimation of the average score (importance) for particles in a given phase-space region. For those who are interested, the details are given on pages 43 and 44 of [2] For the present purpose, one should understand that the generator outputs the reciprocal of the average score generated by particles entering a given phase-space region. A zero is output when there has been no score generated by particles (or their progeny) entering a given phase-space region. There are two reasons that no score is generated from a region

1. No particles have entered the region, hence none can score.
2. Particles have entered the region, but none of those that entered ever scored.

For example, consider the input file:

```
200 cm concrete slab problem
    1  1 -2.03 -2   1
    2  1 -2.03 -3   2
    3  1 -2.03 -4   3
    4  1 -2.03 -5   4
    5  1 -2.03 -6   5
    6  1 -2.03 -7   6
    7  1 -2.03 -8   7
    8  1 -2.03 -9   8
    9  1 -2.03 -10  9
   10  1 -2.03 -11  10
   11  1 -2.03 -12  11
   12  1 -2.03 -13  12
   13  1 -2.03 -14  13
   14  1 -2.03 -15  14
   15  1 -2.03 -16  15
   16  1 -2.03 -17  16
   17  1 -2.03 -18  17
   18  1 -2.03 -19  18
   19  1 -2.03 -20  19
   20  1 -2.03 -21  20
   21  0 -1
   22  0 21

    1 py  0
    2 py  10
    3 py  20
    4 py  30
    5 py  40
    6 py  50
    7 py  60
    8 py  70
    9 py  80
   10 py  90
   11 py  100
   12 py  110
   13 py  120
   14 py  130
   15 py  140
   16 py  150
   17 py  160
   18 py  170
   19 py  180
   20 py  190
   21 py  200
```

```
mode   n
imp:n  1 19r 0 0
c        the following is schaeffer portland concrete
m1
        1001.50c  -.010
        8016.50c  -.529
       11023.51c  -.016
       12000.51c  -.002
       13027.50c  -.034
       14000.51c  -.337
       19000.51c  -.013
       20000.51c  -.044
       26000.55c  -.014
        6012.50c  -.001
sdef   x=0 y=1.e-6 z=0 cel=1 wgt=1  erg=14
c  generate spatial windows
wwg   1 1 .5
f1:n  21
cut:n 1.e20 .01 0 0
ctme   5
nps   100000
prdmp    j -60 j 2
print
```

This input file was run for 100000 particles and the following weight window was generated (print table 200 in the output file).

```
wwp:n 5 3 5 0 0 0
wwe:n  1.0000E+02
wwn1:n    0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
          0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
          0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
          0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
         -1.0000E+00 -1.0000E+00
```

This is not an unusual occurrence, especially for novice users. A look at the tracks entering from print table 126 shows what the trouble is.

1neutron activity in each cell   print table 126

	cell	tracks entering	population	collisions	collisions * weight (per history)	number weighted energy	flux weighted energy	average track weight (relative)	average track mfp (cm)
1	1	110011	100581	169059	1.6906E+00	3.7101E+00	8.8634E+00	1.0000E+00	7.7535E+00
2	2	32444	23197	105918	1.0592E+00	1.5631E+00	5.3249E+00	1.0000E+00	6.5357E+00
3	3	17821	12562	60205	6.0205E-01	1.1263E+00	4.1721E+00	1.0000E+00	6.0578E+00
4	4	9466	6599	33094	3.3094E-01	9.7722E-01	3.6282E+00	1.0000E+00	5.8706E+00
5	5	4822	3408	16760	1.6760E-01	8.8063E-01	3.2999E+00	1.0000E+00	5.7257E+00
6	6	2378	1691	8240	8.2400E-02	8.9272E-01	3.2106E+00	1.0000E+00	5.7490E+00
7	7	1241	845	4564	4.5640E-02	8.4128E-01	3.0275E+00	1.0000E+00	5.6490E+00

8	8	612	419	1952	1.9520E-02	8.4466E-01	2.9016E+00	1.0000E+00	5.7729E+00
9	9	293	202	1084	1.0840E-02	8.5992E-01	2.7081E+00	1.0000E+00	5.7555E+00
10	10	142	100	451	4.5100E-03	8.9456E-01	2.8241E+00	1.0000E+00	5.6081E+00
11	11	61	48	203	2.0300E-03	7.1139E-01	2.4230E+00	1.0000E+00	5.3612E+00
12	12	34	23	125	1.2500E-03	5.2635E-01	2.0172E+00	1.0000E+00	5.0330E+00
13	13	19	13	86	8.6000E-04	5.7828E-01	2.1408E+00	1.0000E+00	5.0071E+00
14	14	9	5	30	3.0000E-04	5.6881E-01	1.7969E+00	1.0000E+00	4.2407E+00
15	15	4	3	14	1.4000E-04	9.2253E-01	3.0257E+00	1.0000E+00	6.7548E+00
16	16	3	1	8	8.0000E-05	1.2307E+00	2.2864E+00	1.0000E+00	7.3662E+00
17	17	2	1	8	8.0000E-05	1.7174E-01	3.0850E-01	1.0000E+00	3.1108E+00
18	18	1	1	2	2.0000E-05	9.2232E-02	9.2974E-02	1.0000E+00	2.9299E+00
19	19	0	0	0	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
20	20	0	0	0	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
total		179363	149699	401803	4.0180E+00				

Note that no particles ever make it past cell 18, thus none cross tally surface 21 and none score.

There are several things that a user might try to solve the problem. There is the obvious solution of just running more particles. How many more? That is a bit difficult to say, but judging by track falloff rate in print table 126, a factor of 10 more particles would probably suffice. This judgment was incorrect when tried (slab1).

In addition to the brute force approach of running more particles, there are some other ways that the problem of zero weight windows sometimes can be successfully approached. Some approaches are:

1. Distribute the source all through the problem so that at least some cells get a good importance estimate. Then run another problem with these importance (weight window) estimates and get better estimates the next time.

   This approach makes use of the fact that an importance function is independent of the source; the importance depends on the tally, not the source.

2. Keep the true source, reduce the density so that some particles score. Then use this window, which should be better than no window, to run a problem at the true density. (Sometimes one may have to take a couple of density steps from the low density runs to the true density runs.

3. Use exponential transform to decrease the effective cross section in a properly weighted way. Keep the true source.

4. Make an initial guess at weight windows or importance function by observing the behavior of the analog calculation. This is often the best way when the number of weight windows or cell importances is a small number. If there are 1000 cells in the problem, this can be quite tedious and time consuming.

5. Use another tally that is highly correlated with the ultimate tally desired, but easier to obtain. Use the generator to optimize the windows for the this other tally. The windows for this other tally, though not optimized for

the desired tally, will at least be better having no information at all. Using the windows for the other tally, it is often possible to generate windows for the desired tally.

The first approach (slab2) generated the window

```
wwp:n 5 3 5 0 0 0
wwe:n  1.0000E+02
wwn1:n    0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
          0.0000E+00  0.0000E+00  7.2866E-04  3.6386E-04  1.8151E-04
          1.2053E-04  4.8785E-05  2.8172E-05  1.0236E-05  4.3976E-06
          2.0247E-06  9.3715E-07  4.5544E-07  2.1243E-07  1.0000E-07
         -1.0000E+00 -1.0000E+00
```

Note from the slab0 run that the particles easily got more than halfway through the slab, thus there should be no problem getting the particles from a source in cell 1 to cell 8 where the weight window information starts. The only other thing to keep in mind is that any particles reaching cell 8 are expected to have a weight of about 0.001 because the lower window bound is 7.2866E-04. Thus, in the slab3 run the weight window from slab2 is used and the source weight is set to 0.001.

Slab3 produces a window in every cell

```
wwp:n 5 3 5 0 0 0
wwe:n  1.0000E+02
wwn1:n    5.0000E-01  1.1527E-01  6.2555E-02  3.2895E-02  1.7015E-02
          8.4250E-03  4.2100E-03  2.0900E-03  1.0050E-03  5.0000E-04
          2.4250E-04  1.1773E-04  5.2577E-05  2.1667E-05  9.5579E-06
          4.7931E-06  1.6319E-06  7.8125E-07  3.6458E-07  1.4757E-07
         -1.0000E+00 -1.0000E+00
```

It may help to insert this window into the input file and iterate once more time because print table 160 indicates that this window is based on only 6 scoring histories, but this demonstration ends because it suffices for the current purpose of showing approach 1 above.

For approach 2, note from slab0 that the particles got at least halfway through the slab. This suggests that if the density were reduced by a factor of 2 (slab4), that some particles would tally and a weight window would be produced. The weight window is not correct for the original density, but at least it is a better window for variance reduction at the true density than a window of all zeroes. Slab4 produces the window

```
wwp:n 5 3 5 0 0 0
wwe:n  1.0000E+02
wwn1:n    5.0000E-01  1.5584E-01  1.1426E-01  8.4810E-02  6.1680E-02
          4.4995E-02  3.2525E-02  2.3585E-02  1.6785E-02  1.1930E-02
          8.4200E-03  5.8800E-03  4.0200E-03  2.7900E-03  1.9600E-03
          1.3950E-03  1.0200E-03  7.1000E-04  4.3000E-04  2.8000E-04
         -1.0000E+00 -1.0000E+00
```

The window produced from slab4 is then input into the problem with the original density (slab5) and then the following window is generated from the slab5 run.

```
wwp:n 5 3 5 0 0 0
wwe:n  1.0000E+02
wwn1:n    5.0000E-01  1.1528E-01  6.2738E-02  3.3215E-02  1.7151E-02
          8.5562E-03  4.2231E-03  2.0497E-03  1.0004E-03  4.8758E-04
          2.3900E-04  1.1243E-04  5.5200E-05  2.5313E-05  1.1992E-05
          5.5469E-06  2.4414E-06  1.0547E-06  4.4922E-07  2.2461E-07
         -1.0000E+00 -1.0000E+00
```

The window may improve with another iteration, but this demonstrates how to get started using approach 2.

Approach 3 uses the exponential transform to reduce the effective cross section in the forward direction. That is, particles preferentially take long jumps in the forward direction. Note that this is different from approach 2 because approach 2 used a fictitious density and approach 3 uses the true density and samples in a biased manner. The particles got at least halfway through in the unbiased problem (slab0), so that the exponential transform is set to reduce the effective cross section in the forward direction by a factor of 2 (slab6). Slab6 generates the warning message “warning. exponential transform usually needs weight window,” but this is not of concern until one needs reliable answers. For example, if the weight windows are unreliable by a factor of 2 from their correct values, it is not a serious problem. But if the final estimates are unreliable by a factor of 2, this is a serious problem. The warning message can be ignored while the user sets up the weight windows. Slab6 produces the window

```
wwp:n 5 3 5 0 0 0
wwe:n  1.0000E+02
wwn1:n    5.0000E-01  1.1458E-01  6.2718E-02  3.3055E-02  1.6950E-02
          8.5871E-03  4.1946E-03  2.0540E-03  1.0088E-03  4.9506E-04
          2.3645E-04  1.1029E-04  4.7777E-05  2.4353E-05  9.8767E-06
          5.1597E-06  2.3141E-06  8.7391E-07  4.3701E-07  2.9811E-07
         -1.0000E+00 -1.0000E+00
```

The window may improve with another iteration, but this demonstrates how to get started using approach 3.

Approach 4 starts by noting from the slab0 run that the tracks entering are decreasing by about a factor of 2 from cell to cell. This suggests that the tracks can be kept relatively constant throughout the problem if the window is set so that the weights in each cell drop by about a factor of two. Using this guess for the window (slab7), slab7 produces the window

```
wwp:n 5 3 5 0 0 0
wwe:n  1.0000E+02
wwn1:n    5.0000E-01  1.1592E-01  6.3000E-02  3.3363E-02  1.7097E-02
          8.5240E-03  4.2772E-03  2.0897E-03  1.0277E-03  4.9595E-04
          2.4282E-04  1.1738E-04  5.6387E-05  2.7182E-05  1.2855E-05
```

```
         5.9984E-06  2.8597E-06  1.3302E-06  6.2955E-07  2.9390E-07
        -1.0000E+00 -1.0000E+00
```

Note that a window can be generated even with guesses that are pretty fair off. For example, slab8 decreased the windows from cell to cell by a factor of 0.707 and produced the window

```
wwp:n 5 3 5 0 0 0
wwe:n  1.0000E+02
wwn1:n    5.0000E-01  1.1593E-01  6.2942E-02  3.3075E-02  1.7017E-02
          8.3292E-03  4.1146E-03  2.0437E-03  9.8542E-04  4.8021E-04
          2.2500E-04  1.0990E-04  5.5729E-05  2.5781E-05  1.1198E-05
          5.8594E-06  2.5391E-06  1.6276E-06  8.1380E-07  4.8828E-07
         -1.0000E+00 -1.0000E+00
```

whereas slab9 decreased the windows from cell to cell by a factor of 0.333 and produced the window

```
wwp:n 5 3 5 0 0 0
wwe:n  1.0000E+02
wwn1:n    5.0000E-01  1.1800E-01  6.2000E-02  3.1500E-02  1.5000E-02
          8.4722E-03  3.9969E-03  1.9918E-03  1.0394E-03  5.2995E-04
          2.6478E-04  1.3001E-04  6.4132E-05  3.1505E-05  1.5440E-05
          7.4715E-06  3.5575E-06  1.6693E-06  7.8905E-07  3.6547E-07
         -1.0000E+00 -1.0000E+00
```

The point here is that the user's guess need not be a great guess in order to begin the process of obtaining a weight window from the generator.

Approach 5 uses the fact that particles that get all the way from the source to the tally must first get part of the way from source to tally. In slab10, the generator is set to produce windows based on a surface tally (f11:n 11, i.e. an intermediate tally at 100cm) that is about midway between source and the tally that is ultimately desired. desired. (Note that a zero importance region has temporarily been set after surface 11; the reason for this will be demonstrated shortly.) Slab10 produces the window

```
wwp:n 5 3 5 0 0 0
wwe:n  1.0000E+02
wwn1:n    5.0000E-01  1.1528E-01  6.2555E-02  3.2895E-02  1.7015E-02
          8.4250E-03  4.2100E-03  2.0900E-03  1.0050E-03  5.0000E-04
         -1.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
          0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
         -1.0000E+00 -1.0000E+00
```

Now, change the -1.0 entry in cell 11 to 0.0 because one no longer wishes the particles to terminate in cell 11. That is,

```
wwp:n 5 3 5 0 0 0
wwe:n  1.0000E+02
```

```
wwn1:n    5.0000E-01  1.1528E-01  6.2555E-02  3.2895E-02  1.7015E-02
          8.4250E-03  4.2100E-03  2.0900E-03  1.0050E-03  5.0000E-04
          0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
          0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
         -1.0000E+00 -1.0000E+00
```

This window from slab10 is then inserted into slab11 and a new window is generated using the original tally (f1:n 21). Slab11 generates the window

```
wwp:n 5 3 5 0 0 0
wwe:n  1.0000E+02
wwn1:n    5.0000E-01  1.1621E-01  6.3145E-02  3.3654E-02  1.7062E-02
          8.5618E-03  4.3042E-03  2.1257E-03  1.0423E-03  5.0584E-04
          2.4569E-04  1.1841E-04  5.5736E-05  2.5203E-05  1.2232E-05
          5.7225E-06  3.0158E-06  1.3830E-06  5.6641E-07  2.3438E-07
         -1.0000E+00 -1.0000E+00
```

Note that through cell 10 the windows generated by slab10 and slab11 are very similar. Because of the very high degree of correlation, the intermediate tally (f11:n 11) in slab10 is acting as a standin for the desired tally (f1:n 21).

Slab12 returns to the question of why slab10 used a zero importance in cell 11. Specifically, slab12 is the same as slab10 except that cell 11 has unit importance. Slab12 generates the window

```
wwp:n 5 3 5 0 0 0
wwe:n  1.0000E+02
wwn1:n    5.0000E-01  1.1528E-01  6.2565E-02  3.2895E-02  1.7015E-02
          8.4250E-03  4.2100E-03  2.0900E-03  1.0100E-03  5.0000E-04
          8.4632E-04  1.5410E-03  1.4517E-03  0.0000E+00  0.0000E+00
          0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
         -1.0000E+00 -1.0000E+00
```

Note that the windows in cells beyond cell 10 are higher than in cell 10. That is, if this window were used as input instead of (slab11's window) then the particles would actually be rouletted as they tried to get from surface 11 (intermediate tally) to surface 21 (desired tally). This is understandable because the generator results for the intermediate tally will naturally have windows that increase as the particles get farther and farther away, in either direction, from surface 11.

3 Zeroes in an Energy-Dependent Weight Window

Although it is often the case that a spatial window can be produced fairly easily, a space and energy dependent window is usually substantially harder. Starting with the spatial window generated by slab11, slab20 generates the energy dependent window

```
wwp:n 5 3 5 0 0 0
wwe:n  1.0000E-01  3.0000E-01  1.0000E+00  2.0000E+00  3.0000E+00
       5.0000E+00  8.0000E+00  1.6000E+01
wwn1:n    0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
          0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
          0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
          0.0000E+00  0.0000E+00  1.7913E-04  1.8089E-05  1.3290E-06
         -1.0000E+00 -1.0000E+00
wwn2:n    0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
          0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
          0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
          0.0000E+00  5.7098E-04  5.5807E-05  5.0814E-06  7.9160E-07
         -1.0000E+00 -1.0000E+00
wwn3:n    0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
          0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
          0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
          0.0000E+00  1.0372E-04  1.4200E-05  2.4648E-06  5.3619E-07
         -1.0000E+00 -1.0000E+00
wwn4:n    0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
          0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
          0.0000E+00  0.0000E+00  9.1766E-03  4.2984E-03  2.9196E-04
          5.9042E-05  1.4474E-05  3.5741E-06  1.0660E-06  3.6739E-07
         -1.0000E+00 -1.0000E+00
wwn5:n    0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
          0.0000E+00  2.7202E-02  1.4535E-02  4.2355E-03  1.6839E-03
          8.6794E-04  4.0984E-04  1.5438E-04  5.4347E-05  2.4817E-05
          1.0141E-05  4.1023E-06  1.6014E-06  6.9490E-07  3.1557E-07
         -1.0000E+00 -1.0000E+00
wwn6:n    3.3333E+00  5.6746E-01  2.7517E-01  1.3336E-01  8.0779E-02
          3.3973E-02  2.1641E-02  1.3506E-02  4.5737E-03  1.7146E-03
          9.7314E-04  4.0262E-04  9.0860E-05  2.9877E-05  1.5873E-05
          7.9925E-06  4.2870E-06  1.3067E-06  6.0036E-07  3.1448E-07
         -1.0000E+00 -1.0000E+00
wwn7:n    2.4824E+00  1.7749E-01  1.2103E-01  5.5493E-02  1.7977E-02
          8.0714E-03  3.8287E-03  1.6807E-03  7.7002E-04  2.3986E-04
          8.9187E-05  3.9580E-05  2.0818E-05  1.0598E-05  6.7783E-06
          2.9829E-06  1.4390E-06  6.7568E-07  4.0832E-07  2.5974E-07
         -1.0000E+00 -1.0000E+00
wwn8:n    5.0000E-01  6.7056E-02  3.0592E-02  1.3709E-02  6.9265E-03
          3.5548E-03  1.6576E-03  7.8467E-04  4.2039E-04  1.6068E-04
          8.9353E-05  4.0319E-05  2.0668E-05  1.0932E-05  5.6682E-06
          2.1028E-06  1.3396E-06  7.2921E-07  4.2023E-07  2.5431E-07
         -1.0000E+00 -1.0000E+00
```

Note that the high energy windows are filled in whereas the low energy windows have lots of zeroes. This is not a surprise because low energy neutrons are much less likely to score using the same energy independent window as the high energy neutrons. Unfortunately, filling in these energy dependent weight windows is more difficult than filling in the zeroes in a spatial window. The goals of this section are to give:

1. An explanation of why it is dangerous to leave zeroes in the windows.

2. An explanation of how to fill in the zeroes.

3. Comments on possible better solutions to this problem in the future.

To see why it is dangerous to leave the zeroes in the weight window, slab21 was run using the energy dependent window from slab20. Here is the fom (figure of merit) table

```
                      tally    1
      nps       mean      error    vov  slope    fom
     4000   2.8688E-07 0.1390 0.0506  0.0     159
     8000   2.6981E-07 0.0988 0.0323  0.0     161
    12000   2.6836E-07 0.0819 0.0239  0.0     158
    16000   2.9351E-07 0.0734 0.0207  0.0     145
    20000   2.8532E-07 0.0652 0.0169 10.0     149
    24000   2.7497E-07 0.0592 0.0145 10.0     152
    28000   2.6941E-07 0.0549 0.0122 10.0     153
    32000   2.7504E-07 0.0518 0.0123 10.0     149
    36000   2.6788E-07 0.0497 0.0117  6.1     145
    40000   2.6369E-07 0.0470 0.0105  5.7     146
    44000   2.7792E-07 0.0629 0.2095  2.5      73
    48000   2.7354E-07 0.0595 0.1973  2.5      75
    50000   2.7331E-07 0.0579 0.1880  2.6      68
```

Note that had the user stopped at 40000 particles, everything in the figure of merit chart would have looked fine. Indeed, it is not shown here, but if one stops at 40000 particles, then the tally passes all ten MCNP statistical checks. (Note that there is some legerdemain in slab21. Specifically, the starting random number was changed so that the fom table would show the problem. Sometimes, the problem never becomes apparent to the user; for example, had the present calculation been stopped at 40000 particles.)

When one sees such strange statistical behavior, it is advisable to get an event log for the particle that gave the largest score, because there will typically be something quite special about that particle. (If there were nothing special about the particle, then it is unlikely that the fom table would look so strange.) At this point, one looks at the output file (slab21.o here) and finds the line (print table 160)

```
history number of largest  tally  =       42000
largest  unnormalized history tally = 5.11163E-04
```

To reproduce this particle history, one would normally use the MCNP input card

```
rand   hist=42000
```

but because slab21 started with

```
rand   hist=118372
```

slab22 uses the following rand and dbcn cards

```
rand   hist=160371
dbcn   j j 1 1 100000
```

to get an event log of particle 1 through particle 1 (i.e., 1 1 on dbcn card above). (Note for MCNP team. The fifth entry on the dbcn card has a default of 600 lines for the event log. This may have been reasonable when the output was printed to paper, but it seems unreasonably small today.) Table 160 for slab22 then gives

```
history number of largest  tally  =           1
largest  unnormalized history tally = 5.11163E-04
```

for a check that the particle history is being reproduced.

Now look at the tracks that tally in the event log. Note that the tracks that tally (for this problem) all have "escape" on the event line print and all have 2.000+02 (the y coordinate upon tallying) in columns 21 to 29. When the text editor is instructed to find every line that has "escape" in it and has 2.000+02 in columns 21 to 29 the result is

```
 *dop*/escape/tp,,21,29/2.000+02
 1317    t    22 -1.871+01  2.000+02 -9.185+01  6.539-02  5.612-01 -8.251-01 7.926-01 1.286-06   escape                  6634
 1376    t    22 -3.571+01  2.000+02 -6.708+01  2.827-01  9.005-01  3.304-01 1.187+00 1.286-06   escape                  6950
 2173    t    22  8.368+00  2.000+02 -3.053+01 -6.390-01  7.292-01 -2.448-01 1.088+00 1.286-06   escape                  12985
 2178    t    22  1.113+01  2.000+02 -3.200+01 -2.889-01  8.175-01  4.982-01 1.309+00 1.286-06   escape                  12994
 2219    t    22  1.309+01  2.000+02 -4.238+01  4.631-01  8.003-01  3.809-01 1.652-01 1.286-06   escape                  13294
 2262    t    22  2.079+01  2.000+02 -4.887+01  1.573-01  9.865-01 -4.616-02 2.354+00 1.286-06   escape                  13599
 2266    t    22  2.079+01  2.000+02 -4.887+01  1.573-01  9.865-01 -4.616-02 2.354+00 1.286-06   escape                  13600
 2276    t    22  1.953+01  2.000+02 -4.911+01  5.142-01  7.470-01  4.215-01 1.798+00 1.286-06   escape                  13642
 2280    t    22  2.079+01  2.000+02 -4.887+01  1.573-01  9.865-01 -4.616-02 2.354+00 1.286-06   escape                  13643
 2515    t    22  1.321+01  2.000+02 -7.452+01  7.012-01  6.602-01  2.689-01 1.162+00 1.286-06   escape                  14974
 2531    t    22 -3.495+00  2.000+02 -7.951+01  1.263-01  9.644-01  2.324-01 2.134+00 1.286-06   escape                  15022
 2696    t    22  1.254+01  2.000+02 -2.895+01 -5.254-01  8.442-01 -1.058-01 6.794-01 1.286-06   escape                  15994
 2848    t    22 -4.835+01  2.000+02 -5.132+01  1.075-01  6.523-01 -7.503-01 9.626-02 2.662-06   escape                  16923
 2857    t    22 -5.473+01  2.000+02 -4.412+01  1.032-01  8.753-01  4.725-01 1.408-01 2.662-06   escape                  16971
 2865    t    22 -5.263+01  2.000+02 -5.885+01 -3.770-01  9.122-01  1.606-01 4.814-01 2.662-06   escape                  17012
 2884    t    22 -4.874+01  2.000+02 -4.743+01 -2.263-01  9.702-01  8.663-02 1.422-02 2.662-06   escape                  17167
 2898    t    22 -6.673+01  2.000+02 -4.541+01 -3.375-01  9.413-01  6.386-03 1.127+00 1.286-06   escape                  17239
 2908    t    22 -5.213+01  2.000+02 -4.484+01  5.758-01  5.713-01 -5.848-01 5.837-01 1.286-06   escape                  17301
 3870    t    22 -5.082+01  2.000+02 -5.706+01  6.731-01  7.297-01 -1.199-01 1.394+00 1.286-06   escape                  24346
 3978    t    22 -2.854+01  2.000+02 -4.659+01  2.573-02  9.954-01  9.281-02 2.364+00 1.286-06   escape                  24981
 3999    t    22 -1.796+01  2.000+02 -8.932+01 -3.588-02  6.135-01  7.889-01 6.730-01 1.286-06   escape                  25104
 4007    t    22 -7.401+00  2.000+02 -8.823+01  2.061-01  7.805-01 -5.903-01 1.995-01 1.286-06   escape                  25153
 4224    t    22 -3.612+01  2.000+02 -2.735+01 -1.687-02  9.975-01  6.923-02 2.148+00 1.286-06   escape                  26622
 4243    t    22 -3.908+01  2.000+02 -2.941+01  2.593-01  9.532-01  1.553-01 2.179+00 1.286-06   escape                  26751
 4249    t    22 -4.945+01  2.000+02 -3.037+01  7.399-01  4.587-01 -4.921-01 5.176-01 1.286-06   escape                  26775
 4258    t    22 -4.021+01  2.000+02 -1.370+01  2.084-01  8.411-01  4.992-01 1.526+00 1.286-06   escape                  26815
 4262    t    22 -4.391+01  2.000+02  1.390+00 -8.516-02  3.691-01  9.255-01 2.246+00 1.286-06   escape                  26816
 4361    t    22 -4.279+01  2.000+02 -4.651+01 -4.521-01  8.866-01  9.776-02 2.353+00 1.286-06   escape                  27470
 4365    t    22 -4.279+01  2.000+02 -4.651+01 -4.521-01  8.866-01  9.776-02 2.353+00 1.286-06   escape                  27471
 4375    t    22 -3.911+01  2.000+02 -4.113+01  6.993-01  6.057-01 -3.796-01 3.390-01 1.286-06   escape                  27544
 4380    t    22 -3.728+01  2.000+02 -4.577+01 -3.394-01  8.549-01  3.923-01 2.347+00 1.286-06   escape                  27553
 4386    t    22 -3.719+01  2.000+02 -4.765+01 -5.103-01  8.576-01  6.426-02 2.349+00 1.286-06   escape                  27563
 4836    t    22 -2.430+01  2.000+02 -5.431+01  9.565-02  9.949-01  3.314-02 2.138+00 1.286-06   escape                  31063
```

```
4849    t   22 -3.005+01  2.000+02 -7.247+01 -2.052-01  4.596-01 -8.641-01 1.621+00 1.286-06   escape            31131
4911    t   22 -2.075+00  2.000+02 -4.966+01  6.162-01  7.276-01  3.014-01 3.034-01 1.286-06   escape            31541
5000    t   22 -9.106+00  2.000+02 -7.014+01  2.460-02  8.525-01 -5.222-01 1.266+00 1.715-06   escape            32125
5064    t   22 -2.417+01  2.000+02 -3.466+01  2.824-01  8.111-01  5.122-01 5.625-01 1.286-06   escape            32560
5070    t   22 -3.531+01  2.000+02 -4.087+01 -7.285-01  6.836-01  4.478-02 1.242+00 1.286-06   escape            32577
5079    t   22 -3.500+00  2.000+02 -4.544+01  3.181-01  9.007-01  2.959-01 1.385+00 1.286-06   escape            32638
5169    t   22 -2.363+01  2.000+02 -5.173+01 -1.536-01  9.881-01  4.301-03 1.724+00 1.286-06   escape            33195
5187    t   22 -3.065+01  2.000+02 -5.466+01 -7.462-01  6.639-01 -4.992-02 9.001-01 1.286-06   escape            33314
5686    t   22 -1.089+01  2.000+02 -1.622+01  6.566-02  8.495-01  5.235-01 2.349+00 1.286-06   escape            37179
5690    t   22 -1.089+01  2.000+02 -1.622+01  6.566-02  8.495-01  5.235-01 2.349+00 1.286-06   escape            37180
5710    t   22 -1.675+01  2.000+02 -4.487-01  4.110-01  5.154-01  7.519-01 2.486-01 1.286-06   escape            37333
5715    t   22 -2.261+01  2.000+02 -1.139+01 -7.380-01  6.689-01 -8.883-02 7.674-01 1.286-06   escape            37351
5723    t   22 -8.725+00  2.000+02 -1.685+01  5.771-01  5.345-01  6.174-01 1.570+00 1.286-06   escape            37379
5727    t   22 -1.089+01  2.000+02 -1.622+01  6.566-02  8.495-01  5.235-01 2.349+00 1.286-06   escape            37380
5733    t   22 -1.089+01  2.000+02 -1.622+01  6.566-02  8.495-01  5.235-01 2.349+00 1.286-06   escape            37383
5737    t   22 -1.089+01  2.000+02 -1.622+01  6.566-02  8.495-01  5.235-01 2.349+00 1.286-06   escape            37384
5745    t   22 -8.156+00  2.000+02 -2.030+01  6.829-01  7.304-01  9.500-03 5.154-01 2.572-06   escape            37423
5762    t   22 -1.028+01  2.000+02 -4.584+01 -1.717-01  6.741-01 -7.184-01 1.541+00 1.286-06   escape            37499
5793    t   22 -5.877+00  2.000+02  1.071+00  1.842-02  9.389-01  3.436-01 3.395-01 1.286-06   escape            37698
5797    t   22 -1.089+01  2.000+02 -1.622+01  6.566-02  8.495-01  5.235-01 2.349+00 1.286-06   escape            37699
5844    t   22 -3.549+01  2.000+02 -3.147+01  4.520-01  1.433-01  8.804-01 7.678-01 1.286-06   escape            38024
5864    t   22 -7.404+00  2.000+02 -1.433+01  4.213-02  6.348-01  7.715-01 8.370-01 1.286-06   escape            38134
5902    t   22 -7.678+00  2.000+02 -2.313+01 -1.660-01  9.571-01  2.373-01 1.101+00 1.286-06   escape            38319
5911    t   22 -2.373+01  2.000+02 -2.385+01 -7.466-01  6.642-01  3.796-02 2.093+00 1.286-06   escape            38370
5916    t   22 -1.914+01  2.000+02 -2.026+01 -4.309-01  6.431-01  6.331-01 1.891+00 1.286-06   escape            38386
5969    t   22 -2.392+01  2.000+02 -3.205+01  9.133-01  2.133-01  3.470-01 1.021-01 2.143-06   escape            38721
5974    t   22 -9.283+00  2.000+02 -5.545+01 -1.229-01  4.818-01 -8.676-01 1.216+00 1.286-06   escape            38730
5995    t   22 -1.794+01  2.000+02 -2.769+01  5.329-01  8.214-01  2.032-01 1.638-02 3.987-06   escape            38924
6000    t   22 -6.712+00  2.000+02 -2.347+01  2.481-01  9.344-01  2.556-01 1.407+00 1.286-06   escape            38926
6009    t   22 -5.566+00  2.000+02 -1.983+01  1.188-02  8.349-01  5.503-01 6.459-01 1.286-06   escape            38974
6023    t   22  4.477+00  2.000+02 -3.021+01  7.262-01  6.123-01 -3.126-01 7.684-01 1.286-06   escape            39064

etcetera
```

Note that some energies (columns 71-80) occur frequently among scoring particles. Note further that almost all of the repeating energies are less than 3 MeV. A glance at the weight window (print table 20 below) shows that above 3 MeV all the space energy regions have nonzero windows.

weight-window lower bounds

energy: cell	1.000E-01	3.000E-01	1.000E+00	2.000E+00	3.000E+00	5.000E+00	8.000E+00	1.000E+36
1	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	3.333E+00	2.482E+00	5.000E-01
2	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	5.675E-01	1.775E-01	6.706E-02
3	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	2.752E-01	1.210E-01	3.059E-02
4	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	1.334E-01	5.549E-02	1.371E-02
5	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	8.078E-02	1.798E-02	6.927E-03
6	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	3.397E-02	8.071E-03	3.555E-03
7	0.000E+00	0.000E+00	0.000E+00	0.000E+00	2.720E-02	2.164E-02	3.829E-03	1.658E-03
8	0.000E+00	0.000E+00	0.000E+00	0.000E+00	1.453E-02	1.351E-02	1.681E-03	7.847E-04
9	0.000E+00	0.000E+00	0.000E+00	0.000E+00	4.235E-03	4.574E-03	7.700E-04	4.204E-04
10	0.000E+00	0.000E+00	0.000E+00	0.000E+00	1.684E-03	1.715E-03	2.399E-04	1.607E-04
11	0.000E+00	0.000E+00	0.000E+00	0.000E+00	8.679E-04	9.731E-04	8.919E-05	8.935E-05
12	0.000E+00	0.000E+00	0.000E+00	0.000E+00	4.098E-04	4.026E-04	3.958E-05	4.032E-05
13	0.000E+00	0.000E+00	0.000E+00	9.177E-03	1.544E-04	9.086E-05	2.082E-05	2.067E-05
14	0.000E+00	0.000E+00	0.000E+00	4.298E-03	5.435E-05	2.988E-05	1.060E-05	1.093E-05
15	0.000E+00	0.000E+00	0.000E+00	2.920E-04	2.482E-05	1.587E-05	6.778E-06	5.668E-06
16	0.000E+00	0.000E+00	0.000E+00	5.904E-05	1.014E-05	7.993E-06	2.983E-06	2.103E-06
17	0.000E+00	5.710E-04	1.037E-04	1.447E-05	4.102E-06	4.287E-06	1.439E-06	1.340E-06
18	1.791E-04	5.581E-05	1.420E-05	3.574E-06	1.601E-06	1.307E-06	6.757E-07	7.292E-07
19	1.809E-05	5.081E-06	2.465E-06	1.066E-06	6.949E-07	6.004E-07	4.083E-07	4.202E-07
20	1.329E-06	7.916E-07	5.362E-07	3.674E-07	3.156E-07	3.145E-07	2.597E-07	2.543E-07
21	-1.000E+00	-1.000E+00	-1.000E+00	-1.000E+00	-1.000E+00	-1.000E+00	-1.000E+00	-1.000E+00
22	-1.000E+00	-1.000E+00	-1.000E+00	-1.000E+00	-1.000E+00	-1.000E+00	-1.000E+00	-1.000E+00

Looking at the first few lines of the event log, one sees that there is a lot of splitting going on (npa=4 indicating a 5:1 split with four tracks banked) at random number indices 157 to 160 as the tracks enter cells 7-9.

```
1     event log for particle history no.        1          ijk =      29601896213953

     cell     x          y          z          u          v          w       erg      wgt                        nch     nrn

 src    1  0.000+00  1.000-06  0.000+00 -7.072-01  5.246-01 -4.740-01 1.400+01 1.000+00   particle=neutron        2
   c    1 -1.047+01  7.765+00 -7.017+00 -6.643-01  6.961-01 -2.724-01 1.396+01 1.000+00 14000.51c r=   2     1    12
   s    2 -1.260+01  1.000+01 -7.891+00 -6.643-01  6.961-01 -2.724-01 1.396+01 3.333-01   surf=     2   npa= 2    13
   c    2 -1.821+01  1.587+01 -1.019+01 -7.211-01 -6.769-02  6.895-01 2.747+00 3.333-01  8016.50c r=  58     2    21
   c    2 -1.849+01  1.585+01 -9.920+00  2.102-01 -1.898-01  9.591-01 2.586+00 3.333-01  8016.50c r=   2     3    31
   c    2 -1.482+01  1.253+01  6.834+00 -1.401-01  9.164-01  3.748-01 2.434+00 3.333-01 14000.51c r=   2     4    39
   s    3 -1.596+01  2.000+01  9.889+00 -1.401-01  9.164-01  3.748-01 2.434+00 3.333-01   surf=     3   npa= 0    40
   c    3 -1.611+01  2.099+01  1.029+01 -1.940-01 -7.644-01 -6.149-01 2.206+00 3.333-01 19000.51c r=   2     5    48
   s    2 -1.636+01  2.000+01  9.497+00 -1.940-01 -7.644-01 -6.149-01 2.206+00 3.333-01   surf=     3   npa= 0    49
   s    1 -1.890+01  1.000+01  1.453+00 -1.940-01 -7.644-01 -6.149-01 2.206+00 3.333-01   surf=     2   npa= 0    50
   s   21 -2.144+01  0.000+00 -6.591+00 -1.940-01 -7.644-01 -6.149-01 2.206+00 3.333-01   surf=     1   npa= 0    51
   t   21 -2.144+01  0.000+00 -6.591+00 -1.940-01 -7.644-01 -6.149-01 2.206+00 3.333-01   escape                  51

 bnk    2 -1.260+01  1.000+01 -7.891+00 -6.643-01  6.961-01 -2.724-01 1.396+01 3.333-01 n wws split                 13 51
   c    2 -2.050+01  1.828+01 -1.113+01 -6.643-01  6.961-01 -2.724-01 1.396+01 3.333-01 14000.51c r=   2     6    54
   t    2 -2.050+01  1.828+01 -1.113+01 -6.643-01  6.961-01 -2.724-01 1.396+01 3.333-01   capture                 54

 bnk    2 -1.260+01  1.000+01 -7.891+00 -6.643-01  6.961-01 -2.724-01 1.396+01 3.333-01 n wws split                 13 54
   c    2 -1.292+01  1.034+01 -8.022+00 -8.473-01  5.080-01  1.549-01 1.207+01 3.333-01 14000.51c r=  52     7    65
   c    2 -1.499+01  1.157+01 -7.644+00 -8.241-01 -4.400-01  3.567-01 3.390+00 3.333-01  1001.50c r=   2     8    82
   s    1 -1.794+01  1.000+01 -6.368+00 -8.241-01 -4.400-01  3.567-01 3.390+00 8.333+00   surf=     2   npa= 0    85
   c    1 -1.897+01  9.450+00 -5.922+00 -4.867-01 -7.311-01  4.782-01 3.345+00 8.333+00  8016.50c r=   2     9    97
   c    1 -2.063+01  6.958+00 -4.292+00 -2.867-01  9.511-01 -1.146-01 2.729+00 8.333+00  8016.50c r=   2    10    105
   s    2 -2.155+01  1.000+01 -4.658+00 -2.867-01  9.511-01 -1.146-01 2.729+00 8.333+00   surf=     2   npa= 0    106
   c    2 -2.183+01  1.095+01 -4.773+00 -3.477-01  4.215-01 -8.375-01 2.651+00 8.333+00 14000.51c r=   2    11    114
   c    2 -2.422+01  1.384+01 -1.052+01 -8.502-02  5.971-01 -7.977-01 2.645+00 8.333+00 20000.51c r=   2    12    122
   c    2 -2.454+01  1.609+01 -1.353+01 -2.001-02  5.858-01 -8.102-01 2.633+00 8.333+00  1001.50c r=   2    13    137
   s    3 -2.467+01  2.000+01 -1.893+01 -2.001-02  5.858-01 -8.102-01 2.633+00 8.333+00   surf=     3   npa= 0    138
   s    4 -2.501+01  3.000+01 -3.276+01 -2.001-02  5.858-01 -8.102-01 2.633+00 8.333+00   surf=     4   npa= 0    139
   c    4 -2.527+01  3.741+01 -4.301+01 -6.239-01  5.079-01 -5.939-01 2.594+00 8.333+00 14000.51c r=   2    14    147
   s    5 -2.845+01  4.000+01 -4.604+01 -6.239-01  5.079-01 -5.939-01 2.594+00 8.333+00   surf=     5   npa= 0    148
   c    5 -4.064+01  4.993+01 -5.765+01  2.774-01  9.605-01  2.076-02 2.375+00 8.333+00  8016.50c r=   2    15    156
   s    6 -4.062+01  5.000+01 -5.765+01  2.774-01  9.605-01  2.076-02 2.375+00 8.333+00   surf=     6   npa= 0    157
   s    7 -3.774+01  6.000+01 -5.743+01  2.774-01  9.605-01  2.076-02 2.375+00 1.667+00   surf=     7   npa= 4    158
   s    8 -3.485+01  7.000+01 -5.722+01  2.774-01  9.605-01  2.076-02 2.375+00 3.333-01   surf=     8   npa= 4    159
   s    9 -3.196+01  8.000+01 -5.700+01  2.774-01  9.605-01  2.076-02 2.375+00 6.667-02   surf=     9   npa= 4    160

   etcetera
```

Why is there so much splitting occurring? Note from the weight window, that the tracks being split have not had their weight checked from the time the progenitor track was in cell 1 (random number index 105; the 'nrn' column) until the progenitor track enters cell 7 because cells 1 to 6 have zeroes for the weight window in the 2 to 3 MeV range. The progenitor track enters cell 7 with weight 8.333 whereas the window in cell 7 is between .0272 and $5 \times .0272$. That is, the progenitor's weight is 61 times the upper bound in the cell 7 and energy range 2 to 3 MeV. With a text editor one finds out that the energy "2.375+00" occurs in columns 71 to 80 of the event log 556 times. It is likely that almost all of these tracks result from the single progenitor track at random number index 105. This should not be occurring. The whole idea of splitting is that the split tracks tend to do different things. Here, there are large numbers of tracks with exactly the same energy.

Note that despite the fact that higher energy particles penetrate the concrete better, one does not see the variance problem in the higher energy ranges. One sees the variance problem in the highest energy range where the weight windows are not controlling the weights. The solution is to fill in the weight window zeroes with some "reasonable values."

So what is a "reasonable" value or, perhaps more appropriately, what is an "unreasonable" value? To answer this question, it helps to understand why the weight window generator produced a zero window in the first place. Note that a zero window means that no tracks ever scored from the phase-space region. This is either because 1) no tracks ever entered the region or because 2) no entering tracks ever scored. In both cases, the space energy region tends to be relatively unimportant to the desired tally. Because of this, one can pick almost any value that is not outrageous. Examples will follow soon.

Before trying some actual weight window numbers, perhaps the essence of the term "reasonable values" can be illustrated by a simple example with binary distributions. Suppose the sampling space is partitioned into two regions R_1 and R_2. Suppose that R_i is chosen with probability c_i. Once in region i, the particle scores s_i with probability p_i and scores 0 with probability $1 - p_i$. The mean for this problem is

$$M_1 = c_1 p_1 s_1 + c_1(1 - p_1)0 + c_2 p_2 s_2 + c_2(1 - p_2)0 = c_1 p_1 s_1 + c_2 p_2 s_2$$

The second moment is

$$M_2 = c_1 p_1 s_1^2 + c_1(1 - p_1)0^2 + c_2 p_2 s_2^2 + c_2(1 - p_2)0^2 = c_1 p_1 s_1^2 + c_2 p_2 s_2^2$$

hence the variance is

$$V = M_2 - M_1^2 = c_1 p_1 s_1^2 + c_2 p_2 s_2^2 - (c_1 p_1 s_1 + c_2 p_2 s_2)^2$$

Suppose, similar to the MCNP example, that most samples of R_2 result in no scoring particle, but that an occasional sample of R_2 results in a large score s_2. This is a high variance situation. Let's put in some numbers now. Let

1. $c_1 = .7$
2. $p_1 = .01$
3. $s_1 = 1$
4. $c_2 = .3$
5. $p_2 = 1.e - 7$
6. $s_2 = 1000$

The mean and variance for this case are $M_1 = 0.00703$ and $V = .0369506$. Note that until a particle scores $s2 = 1000$ from R_2, the sample mean and variance, $\hat{M}_1$ and $\hat{V}$, will be missing any terms involving s_2 and will therefore be approximately

$$\hat{M}_1 \approx c_1 p_1 s_1 = 0.007$$

$$\hat{V} \approx c_1 p_1 s_1^2 - (c_1 p_1 s_1)^2 = .006951$$

Note that the mean is pretty close to the true mean, but the variance estimate will be about a factor of 5 low (.0369506/.006951) until one of the large $s_2 = 1000$ scores occurs. Suppose that a variance reduction attack is mounted to increase the probability that a particle in R_2 scores, and correspondingly decrease the magnitude of the score when the particle does score. Assume that the variance reduction method applied preserves the binomial distribution of scores from R_2, but changes the binomial p_2. Note that M_1 will be preserved as long as the product $p_2 s_2$ is preserved. For the specific case under consideration then $p_2 s_2 = 1. \times 10^{-7} 1000 = 1. \times 10^{-4}$ Thus, p_2 can be adjusted to anything and M_1 will be preserved, if s_2 is correspondingly adjusted to $s_2 = .0001/p_2$. Some possible adjustments of p_2 and s_2 are shown in the following table.

p2	s2	variance
1.e-9	100000	3.00695
1.e-8	10000	.306951
1.e-7	1000	.0369506
1.e-6	100	.00995058
1.e-5	10	.00725058
1.e-4	1	.00698058
1.e-3	.1	.00695358
1.e-2	.01	.00695088
1.e-1	.001	.00695061
1.e+0	.0001	.00695058

Note that the last entry, with $p_2 = 1$, is absolutely the best that can be done for particles entering R_2 since *every* particle arriving in R_2 scores *exactly* .0001; that is, a zero variance sampling of the problem occurs after arriving in R_2. Note however, that *any* $.00001 \leq p_2 \leq 1$ gives approximately the same value for the variance. Thus, there is a very broad range of reasonable values for p_2 that remove the variance problem initially encountered. For this reason, the user should not fret unduly about using a "good" choice of variance reduction parameters; he merely has to avoid "bad" choices.

Returning to the actual weight window at hand, one can note the general behavior in the weight window. For the highest energy window, the windows are increasing by something like a factor of 2 for each cell moving away from the tally region. For the lowest energy window he windows are increasing by something like a factor of 10 for each cell moving away from the tally region. Note that having high weight windows saves time relative to lower weight windows because their is less splitting and/or more roulette. Conversely, having high windows tends to increase the history variance compared with having low windows. Thus, from a conservative point of view, one is much less likely to get into variance problems by using a factor of 2 rule to extend the window into regions where the generator supplied a zero. (It is convenient to figure what the window value in cell 1 is and then use a 0.5m multiplier entry to multiply each subsequent entry by 0.5. Note for MCNP code developers: it might be nice to allow a divide option as well as a multiply option.)

```
wwp:n 5 3 5 0 0 0
wwe:n  1.0000E-01  3.0000E-01  1.0000E+00  2.0000E+00  3.0000E+00
       5.0000E+00  8.0000E+00  1.6000E+01
wwn1:n    23.4789     0.5m        0.5m        0.5m        0.5m
          0.5m        0.5m        0.5m        0.5m        0.5m
          0.5m        0.5m        0.5m        0.5m        0.5m
          0.5m        0.5m        1.7913E-04  1.8089E-05  1.3290E-06
         -1.0000E+00 -1.0000E+00
wwn2:n    37.4197     0.5m        0.5m        0.5m        0.5m
          0.5m        0.5m        0.5m        0.5m        0.5m
          0.5m        0.5m        0.5m        0.5m        0.5m
          0.5m        5.7098E-04  5.5807E-05  5.0814E-06  7.9160E-07
         -1.0000E+00 -1.0000E+00
wwn3:n    6.79739     0.5m        0.5m        0.5m        0.5m
          0.5m        0.5m        0.5m        0.5m        0.5m
          0.5m        0.5m        0.5m        0.5m        0.5m
          0.5m        1.0372E-04  1.4200E-05  2.4648E-06  5.3619E-07
         -1.0000E+00 -1.0000E+00
wwn4:n    37.5874     0.5m        0.5m        0.5m        0.5m
          0.5m        0.5m        0.5m        0.5m        0.5m
          0.5m        0.5m        9.1766E-03  4.2984E-03  2.9196E-04
          5.9042E-05  1.4474E-05  3.5741E-06  1.0660E-06  3.6739E-07
         -1.0000E+00 -1.0000E+00
wwn5:n    1.74093     0.5m        0.5m        0.5m        0.5m
          0.5m        2.7202E-02  1.4535E-02  4.2355E-03  1.6839E-03
          8.6794E-04  4.0984E-04  1.5438E-04  5.4347E-05  2.4817E-05
          1.0141E-05  4.1023E-06  1.6014E-06  6.9490E-07  3.1557E-07
         -1.0000E+00 -1.0000E+00
wwn6:n    3.3333E+00  5.6746E-01  2.7517E-01  1.3336E-01  8.0779E-02
          3.3973E-02  2.1641E-02  1.3506E-02  4.5737E-03  1.7146E-03
          9.7314E-04  4.0262E-04  9.0860E-05  2.9877E-05  1.5873E-05
          7.9925E-06  4.2870E-06  1.3067E-06  6.0036E-07  3.1448E-07
         -1.0000E+00 -1.0000E+00
wwn7:n    2.4824E+00  1.7749E-01  1.2103E-01  5.5493E-02  1.7977E-02
          8.0714E-03  3.8287E-03  1.6807E-03  7.7002E-04  2.3986E-04
          8.9187E-05  3.9580E-05  2.0818E-05  1.0598E-05  6.7783E-06
          2.9829E-06  1.4390E-06  6.7568E-07  4.0832E-07  2.5974E-07
         -1.0000E+00 -1.0000E+00
wwn8:n    5.0000E-01  6.7056E-02  3.0592E-02  1.3709E-02  6.9265E-03
          3.5548E-03  1.6576E-03  7.8467E-04  4.2039E-04  1.6068E-04
          8.9353E-05  4.0319E-05  2.0668E-05  1.0932E-05  5.6682E-06
          2.1028E-06  1.3396E-06  7.2921E-07  4.2023E-07  2.5431E-07
         -1.0000E+00 -1.0000E+00
```

Or, from Table 20 of the output file (slab23.o)

weight-window lower bounds

energy:	1.000E-01	3.000E-01	1.000E+00	2.000E+00	3.000E+00	5.000E+00	8.000E+00	1.000E+36
cell								
1	2.348E+01	3.742E+01	6.797E+00	3.759E+01	1.741E+00	3.333E+00	2.482E+00	5.000E-01
2	1.174E+01	1.871E+01	3.399E+00	1.879E+01	8.705E-01	5.675E-01	1.775E-01	6.706E-02
3	5.870E+00	9.355E+00	1.699E+00	9.397E+00	4.352E-01	2.752E-01	1.210E-01	3.059E-02
4	2.935E+00	4.677E+00	8.497E-01	4.698E+00	2.176E-01	1.334E-01	5.549E-02	1.371E-02
5	1.467E+00	2.339E+00	4.248E-01	2.349E+00	1.088E-01	8.078E-02	1.798E-02	6.927E-03
6	7.337E-01	1.169E+00	2.124E-01	1.175E+00	5.440E-02	3.397E-02	8.071E-03	3.555E-03
7	3.669E-01	5.847E-01	1.062E-01	5.873E-01	2.720E-02	2.164E-02	3.829E-03	1.658E-03
8	1.834E-01	2.923E-01	5.310E-02	2.937E-01	1.453E-02	1.351E-02	1.681E-03	7.847E-04
9	9.171E-02	1.462E-01	2.655E-02	1.468E-01	4.235E-03	4.574E-03	7.700E-04	4.204E-04
10	4.586E-02	7.309E-02	1.328E-02	7.341E-02	1.684E-03	1.715E-03	2.399E-04	1.607E-04
11	2.293E-02	3.654E-02	6.638E-03	3.671E-02	8.679E-04	9.731E-04	8.919E-05	8.935E-05
12	1.146E-02	1.827E-02	3.319E-03	1.835E-02	4.098E-04	4.026E-04	3.958E-05	4.032E-05

```
13          5.732E-03  9.136E-03  1.660E-03  9.177E-03  1.544E-04  9.086E-05  2.082E-05  2.067E-05
14          2.866E-03  4.568E-03  8.298E-04  4.298E-03  5.435E-05  2.988E-05  1.060E-05  1.093E-05
15          1.433E-03  2.284E-03  4.149E-04  2.920E-04  2.482E-05  1.587E-05  6.778E-06  5.668E-06
16          7.165E-04  1.142E-03  2.074E-04  5.904E-05  1.014E-05  7.993E-06  2.983E-06  2.103E-06
17          3.583E-04  5.710E-04  1.037E-04  1.447E-05  4.102E-06  4.287E-06  1.439E-06  1.340E-06
18          1.791E-04  5.581E-05  1.420E-05  3.574E-06  1.601E-06  1.307E-06  6.757E-07  7.292E-07
19          1.809E-05  5.081E-06  2.465E-06  1.066E-06  6.949E-07  6.004E-07  4.083E-07  4.202E-07
20          1.329E-06  7.916E-07  5.362E-07  3.674E-07  3.156E-07  3.145E-07  2.597E-07  2.543E-07
21         -1.000E+00 -1.000E+00 -1.000E+00 -1.000E+00 -1.000E+00 -1.000E+00 -1.000E+00 -1.000E+00
22         -1.000E+00 -1.000E+00 -1.000E+00 -1.000E+00 -1.000E+00 -1.000E+00 -1.000E+00 -1.000E+00
```

Using this filled in window, the calculation now appears well behaved at 500,000 particles as indicated by MCNP's ten statistical checks and the tally fluctuation chart below.

```
1tally fluctuation charts

                      tally    1
        nps      mean     error   vov  slope    fom
      32000   2.4724E-07 0.0496 0.0104  5.1     293
      64000   2.5002E-07 0.0364 0.0055  5.0     274
      96000   2.5163E-07 0.0297 0.0037 10.0     272
     128000   2.5832E-07 0.0261 0.0042  6.0     261
     160000   2.5450E-07 0.0236 0.0035  5.6     259
     192000   2.5251E-07 0.0216 0.0028  6.4     259
     224000   2.5510E-07 0.0198 0.0023  7.6     261
     256000   2.5781E-07 0.0186 0.0022  8.5     258
     288000   2.5777E-07 0.0176 0.0019 10.0     258
     320000   2.5721E-07 0.0168 0.0021  6.6     254
     352000   2.5921E-07 0.0160 0.0019  5.4     253
     384000   2.5817E-07 0.0153 0.0017  5.0     256
     416000   2.5792E-07 0.0146 0.0015  5.5     257
     448000   2.5838E-07 0.0141 0.0014  6.3     258
     480000   2.5835E-07 0.0135 0.0013  5.7     260
     500000   2.5849E-07 0.0133 0.0012  6.2     260

 ********************************************************************************************************************
 dump no.    2 on file slab23.r     nps =      500000     coll =        8020654     ctm =       21.86   nrn =          84378048
```

To see that there is nothing magic about the choice of a factor of 2, lets use a factor of 3 between windows in adjacent cells instead. This results in the window

```
wwp:n 5 3 5 0 0 0
wwe:n  1.0000E-01  3.0000E-01  1.0000E+00  2.0000E+00  3.0000E+00
       5.0000E+00  8.0000E+00  1.6000E+01
wwn1:n    23129.0     0.3333334m  0.3333334m  0.3333334m  0.3333334m
          0.3333334m  0.3333334m  0.3333334m  0.3333334m  0.3333334m
          0.3333334m  0.3333334m  0.3333334m  0.3333334m  0.3333334m
          0.3333334m  0.3333334m  1.7913E-04  1.8089E-05  1.3290E-06
         -1.0000E+00 -1.0000E+00
wwn2:n    73620.2     0.3333334m  0.3333334m  0.3333334m  0.3333334m
          0.3333334m  0.3333334m  0.3333334m  0.3333334m  0.3333334m
          0.3333334m  0.3333334m  0.3333334m  0.3333334m  0.3333334m
          0.3333334m  5.7098E-04  5.5807E-05  5.0814E-06  7.9160E-07
         -1.0000E+00 -1.0000E+00
wwn3:n    4464.81     0.3333334m  0.3333334m  0.3333334m  0.3333334m
          0.3333334m  0.3333334m  0.3333334m  0.3333334m  0.3333334m
          0.3333334m  0.3333334m  0.3333334m  0.3333334m  0.3333334m
          0.3333334m  1.0372E-04  1.4200E-05  2.4648E-06  5.3619E-07
         -1.0000E+00 -1.0000E+00
wwn4:n    4876.82     0.3333334m  0.3333334m  0.3333334m  0.3333334m
          0.3333334m  0.3333334m  0.3333334m  0.3333334m  0.3333334m
          0.3333334m  0.3333334m  9.1766E-03  4.2984E-03  2.9196E-04
          5.9042E-05  1.4474E-05  3.5741E-06  1.0660E-06  3.6739E-07
         -1.0000E+00 -1.0000E+00
```

```
wwn5:n    19.8303     0.3333334m  0.3333334m  0.3333334m  0.3333334m
          0.3333334m  2.7202E-02  1.4535E-02  4.2355E-03  1.6839E-03
          8.6794E-04  4.0984E-04  1.5438E-04  5.4347E-05  2.4817E-05
          1.0141E-05  4.1023E-06  1.6014E-06  6.9490E-07  3.1557E-07
         -1.0000E+00 -1.0000E+00
wwn6:n    3.3333E+00  5.6746E-01  2.7517E-01  1.3336E-01  8.0779E-02
          3.3973E-02  2.1641E-02  1.3506E-02  4.5737E-03  1.7146E-03
          9.7314E-04  4.0262E-04  9.0860E-05  2.9877E-05  1.5873E-05
          7.9925E-06  4.2870E-06  1.3067E-06  6.0036E-07  3.1448E-07
         -1.0000E+00 -1.0000E+00
wwn7:n    2.4824E+00  1.7749E-01  1.2103E-01  5.5493E-02  1.7977E-02
          8.0714E-03  3.8287E-03  1.6807E-03  7.7002E-04  2.3986E-04
          8.9187E-05  3.9580E-05  2.0818E-05  1.0598E-05  6.7783E-06
          2.9829E-06  1.4390E-06  6.7568E-07  4.0832E-07  2.5974E-07
         -1.0000E+00 -1.0000E+00
wwn8:n    5.0000E-01  6.7056E-02  3.0592E-02  1.3709E-02  6.9265E-03
          3.5548E-03  1.6576E-03  7.8467E-04  4.2039E-04  1.6068E-04
          8.9353E-05  4.0319E-05  2.0668E-05  1.0932E-05  5.6682E-06
          2.1028E-06  1.3396E-06  7.2921E-07  4.2023E-07  2.5431E-07
         -1.0000E+00 -1.0000E+00
```

Table 20 in the output file for slab24 (24.o) gives the explicit window

weight-window lower bounds

energy:	1.000E-01	3.000E-01	1.000E+00	2.000E+00	3.000E+00	5.000E+00	8.000E+00	1.000E+36
cell								
1	2.313E+04	7.362E+04	4.465E+03	4.877E+03	1.983E+01	3.333E+00	2.482E+00	5.000E-01
2	7.710E+03	2.454E+04	1.488E+03	1.626E+03	6.610E+00	5.675E-01	1.775E-01	6.706E-02
3	2.570E+03	8.180E+03	4.961E+02	5.419E+02	2.203E+00	2.752E-01	1.210E-01	3.059E-02
4	8.566E+02	2.727E+03	1.654E+02	1.806E+02	7.345E-01	1.334E-01	5.549E-02	1.371E-02
5	2.855E+02	9.089E+02	5.512E+01	6.021E+01	2.448E-01	8.078E-02	1.798E-02	6.927E-03
6	9.518E+01	3.030E+02	1.837E+01	2.007E+01	8.161E-02	3.397E-02	8.071E-03	3.555E-03
7	3.173E+01	1.010E+02	6.125E+00	6.690E+00	2.720E-02	2.164E-02	3.829E-03	1.658E-03
8	1.058E+01	3.366E+01	2.042E+00	2.230E+00	1.453E-02	1.351E-02	1.681E-03	7.847E-04
9	3.525E+00	1.122E+01	6.805E-01	7.433E-01	4.235E-03	4.574E-03	7.700E-04	4.204E-04
10	1.175E+00	3.740E+00	2.268E-01	2.478E-01	1.684E-03	1.715E-03	2.399E-04	1.607E-04
11	3.917E-01	1.247E+00	7.561E-02	8.259E-02	8.679E-04	9.731E-04	8.919E-05	8.935E-05
12	1.306E-01	4.156E-01	2.520E-02	2.753E-02	4.098E-04	4.026E-04	3.958E-05	4.032E-05
13	4.352E-02	1.385E-01	8.401E-03	9.177E-03	1.544E-04	9.086E-05	2.082E-05	2.067E-05
14	1.451E-02	4.618E-02	2.800E-03	4.298E-03	5.435E-05	2.988E-05	1.060E-05	1.093E-05
15	4.836E-03	1.539E-02	9.335E-04	2.920E-04	2.482E-05	1.587E-05	6.778E-06	5.668E-06
16	1.612E-03	5.131E-03	3.112E-04	5.904E-05	1.014E-05	7.993E-06	2.983E-06	2.103E-06
17	5.373E-04	5.710E-04	1.037E-04	1.447E-05	4.102E-06	4.287E-06	1.439E-06	1.340E-06
18	1.791E-04	5.581E-05	1.420E-05	3.574E-06	1.601E-06	1.307E-06	6.757E-07	7.292E-07
19	1.809E-05	5.081E-06	2.465E-06	1.066E-06	6.949E-07	6.004E-07	4.083E-07	4.202E-07
20	1.329E-06	7.916E-07	5.362E-07	3.674E-07	3.156E-07	3.145E-07	2.597E-07	2.543E-07
21	-1.000E+00	-1.000E+00	-1.000E+00	-1.000E+00	-1.000E+00	-1.000E+00	-1.000E+00	-1.000E+00
22	-1.000E+00	-1.000E+00	-1.000E+00	-1.000E+00	-1.000E+00	-1.000E+00	-1.000E+00	-1.000E+00

After 20 minutes, this calculation also passes the ten statistical checks. Here is the tally fluctuation chart.

1tally fluctuation charts

	tally 1				
nps	mean	error	vov	slope	fom
32000	2.4608E-07	0.0526	0.0126	10.0	288
64000	2.5135E-07	0.0371	0.0066	10.0	287
96000	2.5795E-07	0.0304	0.0047	9.6	282
128000	2.6604E-07	0.0260	0.0037	8.7	283
160000	2.6151E-07	0.0235	0.0030	9.1	280
192000	2.6199E-07	0.0214	0.0024	7.7	282
224000	2.6122E-07	0.0196	0.0020	6.9	288
256000	2.6400E-07	0.0184	0.0017	7.6	286
288000	2.6541E-07	0.0177	0.0029	4.2	273

```
 320000   2.6241E-07 0.0169 0.0026  4.5     272
 352000   2.6166E-07 0.0161 0.0023  4.8     272
 384000   2.5933E-07 0.0154 0.0021  5.1     275
 416000   2.5896E-07 0.0148 0.0020  5.1     272
 448000   2.5884E-07 0.0142 0.0018  5.2     274
 480000   2.5980E-07 0.0137 0.0017  4.9     275
 500000   2.5983E-07 0.0134 0.0016  4.8     276

*************************************************************************************************************************
dump no.    2 on file slab24.r     nps =      500000     coll =        7666762     ctm =       20.14   nrn =          80432811
```

One final note before leaving this topic. Russian roulette games typically increase the history variance and decrease the computer time per history. A roulette game with a survival probability of 0.1 can often save about 90% of the time that would otherwise be spent processing the particle. Changing this roulette game to have a survival probability of 0.01 can often save about 99% of the processing time. Note that the *extra* time saved by using 0.01 rather than 0.1 is only 9%. Meanwhile, the weight of the particle has been increased by a factor of 10 and so any scores that the particle does make will have an impact 10^2 times as great as with the milder roulette game. Thus, the calculation in slab23 is to be preferred over the calculation in slab24. Usually, there is little point in having weight windows that are more than 100 times the source weight. The extra time saved is very small and the possible bad effects of increased weight grow as the windows get larger.

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