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MCNP Calculations of Subcritical Fixed Source and Fission Multiplication Factors

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INTRODUCTION

Accurate modeling of subcritical multiplication is needed for accelerator-driven subcritical reactors, reactor startups, critical experiment approaches to criticality, and detection of nuclear materials. Such calculations are multiplied fixed-source calculations, not eigenvalue calculations. The traditional approach to obtaining the subcritical multiplication factor M, however, is based on the k_{eff} eigenvalue [1]:

$$M_{\text{eff}} = \frac{S+Q}{Q} = \frac{1}{1-k_{\text{eff}}} . \tag{1}$$

where k_{eff} is determined by a standard criticality eigenvalue calculation, Q is the fixed source intensity, and S is the steady state fission production. This formulation is valid only if the fixed source distribution q is distributed in space and energy as the fundamental critical eigenmode. Generally, this is not the case, and in systems, such as those driven by 14 MeV DT fusion neutrons, the actual multiplication may be significantly different.

Multiplication factors that treat the fixed source and fission distribution separately in calculating the subcritical multiplication were developed in [2] and provide a more realistic way to model subcritical multiplication. This summary provides a brief review of subcritical multiplication factors based on [2], prescribes a means of calculating them using Monte Carlo methods, and provides numerical results to validate the approach. Comparison of the numerical results to Eq. (1) is also provided to illustrate the inaccuracies that may arise from Eq. (1).

THEORY & METHOD

A form of the subcritical multiplication accounting for the fixed and fission sources is developed. Next, an overview of the Monte Carlo method implemented in the MCNP5 [3] production Monte Carlo code is given.

Derivation of Subcritical Multiplication

A formal derivation of the subcritical multiplication factors is given in [2] by manipulating the transport equation:

$$H\psi = F\psi + q . \tag{2}$$

The angular flux is ψ, H is the operator for streaming, collisions, and scattering, F is the operator for fission, and q is the external source.

An adjoint equation for the production of fission neutrons in the current generation is also defined:

$$H^{\dagger}\psi^{\dagger} = \nu\Sigma_f(\mathbf{r}, E) . \tag{3}$$

Σ_f is the macroscopic fission cross section, and ν is the average number of neutrons produced per fission. By the process of solving equations (2) and (3) in [2], it is now possible to define the subcritical multiplication factors for fission and external source neutrons:

$$k_f = \frac{\langle \psi^{\dagger}, F\psi \rangle}{S} , \tag{4}$$

$$k_q = \frac{\langle \psi^{\dagger}, q \rangle}{Q} . \tag{5}$$

A more accurate equation for the subcritical multiplication is obtained:

$$M = \frac{S+Q}{Q} = 1 + \frac{k_q}{1-k_f} . \tag{6}$$

The subcritical source multiplication factor k_q corresponds to how the source distribution multiplies instantaneously. The subcritical fission multiplication factor k_f measures the multiplication from the neutrons in all subsequent generations. Note that for the special case where the external source is the fundamental eigenmode, $k_q = k_f = k_{\text{eff}}$, equation (6) reverts to equation (1).

While both of these depend upon the system properties and the source, separating these two quantities can be useful in understanding how both the external source and fission contribute to the overall multiplication.

Monte Carlo Implementation

A method to compute k_f, k_q, and M is implemented in MCNP5 v1.5.1 [4].

Computing S and Q involve summing the particle weights of all external source or fission neutron production events respectively. Calculating the integrals (tallies) in the numerators of equations (4) and (5) can be done without solving the adjoint equation in (3). The physical meaning of (3) is the expected neutron production in the current generation for a source point (whether external or from fission) at position $\mathbf{r}$ and energy E. If P neutrons are produced during absorption, the tallies take the form:

$$\left\langle \psi^{\dagger}, F\psi \right\rangle = \frac{1}{W}\sum_{\mathrm{f}} wP \,, \tag{7}$$

$$\left\langle \psi^{\dagger}, q \right\rangle = \frac{1}{W}\sum_{\mathrm{q}} wP \,. \tag{8}$$

The summations are over all fission events in all histories resulting from a neutron that was originally produced from fission or from an external source respectively. The weight of the particle at the fission production event is w, and the total source weight in all histories is W.

Dividing the tallies in (7) and (8) by S and Q respectively will give the multiplication factors k_f and k_q.

VERFICATION & RESULTS

First, a simple multigroup slab problem is run in both MCNP and the discrete ordinates code Partisn [5] to compare the subcritical multiplication. Then, results with continuous-energy data [6] are obtained for various point sources in a subcritical assembly consisting of a high enriched uranium (HEU) sphere surrounded by a depleted uranium (DU) reflector.

Discrete Ordinates Comparison

A 1-D bare slab problem (thickness 20 cm) is run with a spatially uniform source in energy group 1. The 4-group cross section data are given in Table I. To facilitate comparison, a very fine space-angle resolution in Partisn is used.

Table I. Cross section data (barns) for the bare slab test problem (N = 0.01 atoms/b-cm).

g	σ_a	$\nu\sigma_f$	σ_t	χ	σ_{sg1}	σ_{sg2}	σ_{sg3}	σ_{sg4}
1	3.0	9.6	5.0	0.0	0.5	0.5	0.5	0.5
2	2.0	5.4	5.0	0.2	0.0	1.0	0.5	0.5
3	2.0	5.2	5.0	0.8	0.0	0.0	1.5	0.5
4	3.0	2.5	5.0	0.0	0.0	0.0	0.0	2.0

Table II. Verification comparing the Monte Carlo method results to those obtained with a direct calculation with discrete ordinates fluxes.

	MCNP	Partisn	% Error
k_{eff}	0.82739 +/- 0.00011	0.82757	-0.022
k_f	0.82694 +/- 0.00009	0.82686	+0.010
k_q	1.38156 +/- 0.00049	1.38232	-0.055
M	8.98328 +/- 0.00293	8.98377	-0.005

The results for the multiplication factors are computed by MCNP5 for Monte Carlo and directly from the output fluxes from Partisn for discrete ordinates. These are compared in Table II and agree within 0.1 percent and are within the 2-σ confidence band.

Reflected HEU Sphere Results

A sphere of HEU (enrichment 93 w/o with radius 4.8 cm) is reflected by a DU (pure ^{238}U) spherical shell with outer radius of 12.2 cm. The sphere has a mass density of 19.1 g/cc. Isotropic point sources are placed starting at the center and going outward in 1 cm increments to 15 cm. The energies of the point sources are a thermal source (monoenergetic 0.025 eV), Watt fission spectrum, and DT fusion source (monoenergetic 14.1 MeV).

The multiplication factors for the fission, k_f, and fixed sources, k_q, are displayed in Figs. 1 and 2 respectively. The fission multiplication is also compared to a reference value for the effective multiplication factor k_{eff}, which is 0.77114 +/- 0.00013.

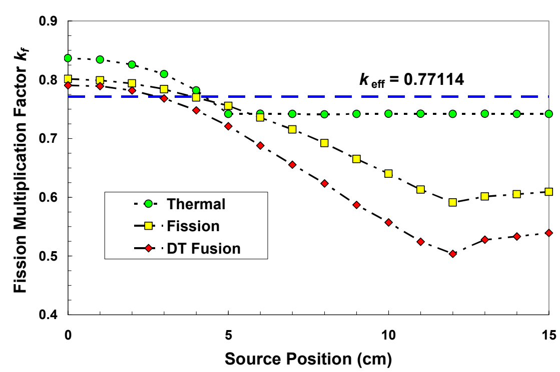

Fig. 1. k_f as a function of source position and energy.

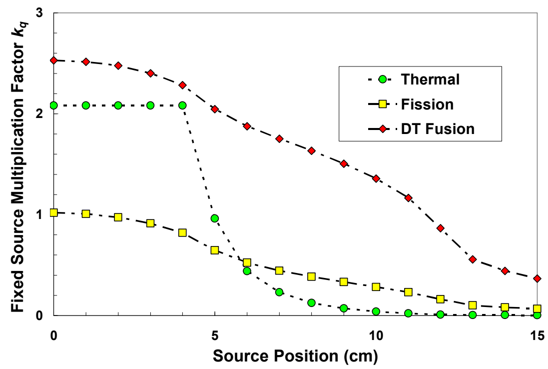

Fig. 2. k_q as a function of source position and energy.

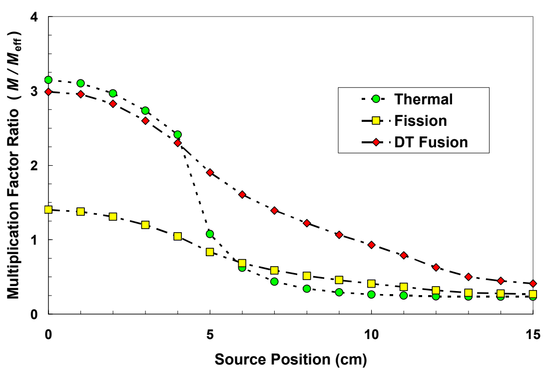

Fig. 3. Ratio of the true multiplication factor to the multiplication factor for the eigenvalue solution as a function of source position and energy.

The multiplication for the fission source k_f is generally not the same as k_{eff} depending upon the source location. For example, a fixed, isotropic DT fusion source outside the assembly would multiply significantly less than the standard eigenvalue approach would predict. Likewise, more centralized sources would give greater multiplication.

The effect of position is particularly pronounced for the source multiplication k_q, especially when the spectrum is significantly different than the fission spectrum.

To illustrate that the eigenvalue solution can be vastly incorrect, the ratio of the true subcritical multiplication to the one predicted by an eigenvalue calculation is shown in Fig. 3. For instance, an isotropic DT fusion source in the center multiplies about three times more than if the fission source is distributed with the fundamental eigenmode.

SUMMARY & CONCLUSIONS

A production Monte Carlo code (MCNP5) can compute multiplication factors for fixed source subcritical multiplication. The Monte Carlo calculations are verified against deterministic estimates with Partisn, and a representative problem (using continuous-energy physics) illustrating the behavior of these factors is shown.

Both Monte Carlo and deterministic methods can currently compute subcritical multiplication using a variety of techniques. What this new approach offers is the decomposition of multiplication for the external source and the resulting fission distribution. Knowledge of these factors gives the designer insight into how to achieve the desired multiplication.

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